Hamiltonian modelling of approximate path planning problems for hybrid algorithms

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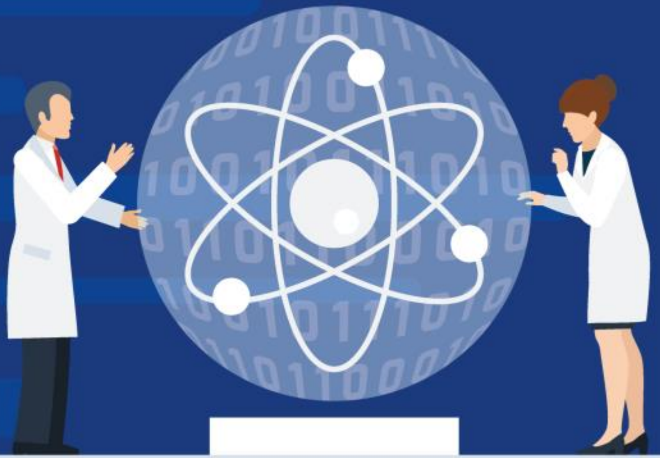

FORUM TERATEC 2021 – TECHNICAL WORKSHOP

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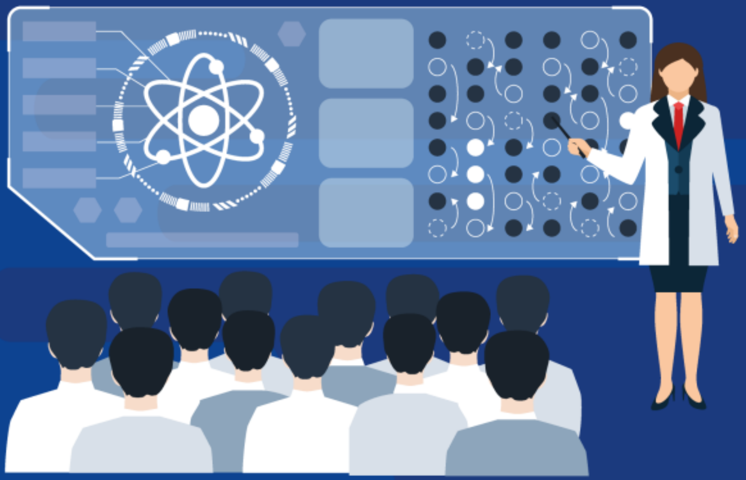
OUTLINE

1. INTRODUCTION
2. COMBINATORIAL OPTIMIZATION PROBLEMS AND QUANTUM METHODS
3. SOLVING A SHORTEST PATH PROBLEM WITH QAOA
4. USING A HEURISTIC METHOD
5. CONCLUSION



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INTRODUCTION

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QUANTUM COMPUTING & NAVAL GROUP

- **A prospective effort**

Preliminary studies launched in 2020 and consolidated in 2021 under the CTO supervision. The first analysis was aiming at identifying use cases and specific algorithms to experiment. NG is currently investigating combinatorial optimization for path planning and for track reconstruction. Additional applications will be tested in 2021/22.

- **Part of a larger program on quantum technologies**

Several topic to address, with different time scales :

→ High sensitivity sensors (EM, gravimeters, IMU) for detection, measurements and navigation.
→ QKD proof-of-concept on board, robust cryptography
→ Superconducting materials for magnetic energy storage (SMES)
→ Corrosion diagnostics using quantum resistive sensors
→ and of course : hybrid algorithms in QC

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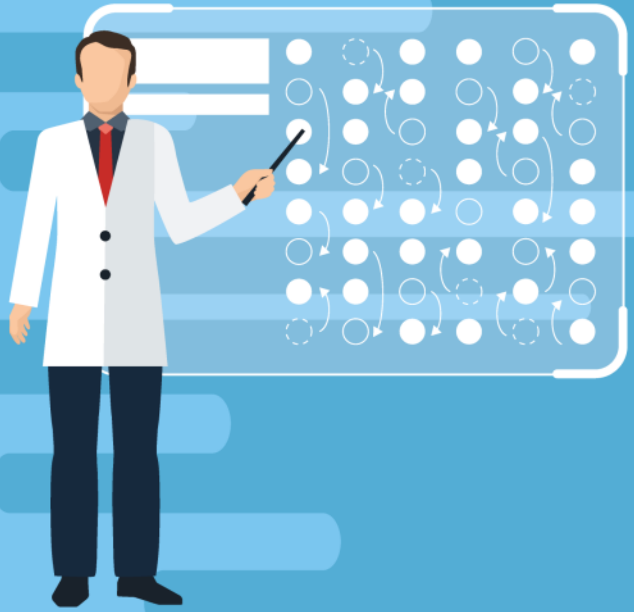
COMBINATORIAL OPTIMIZATION PROBLEMS AND QUANTUM METHODS



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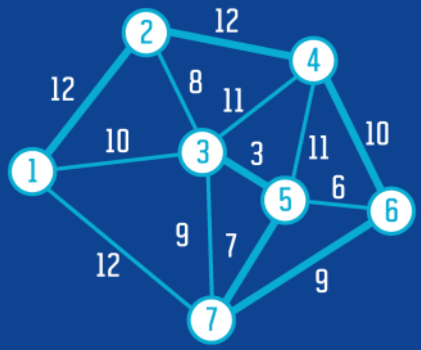
COMBINATORIAL OPTIMIZATION PROBLEMS

THREE PROBLEMS COMMONLY USED AT NAVAL GROUP :

MAXCUT

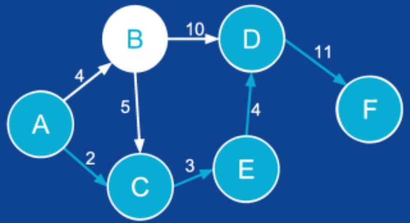
EX : IMAGE PROCESSING

TRAVELLING SALESMAN



EX : DRONE TRAJECTORY PLANNING.

SHORTEST PATH

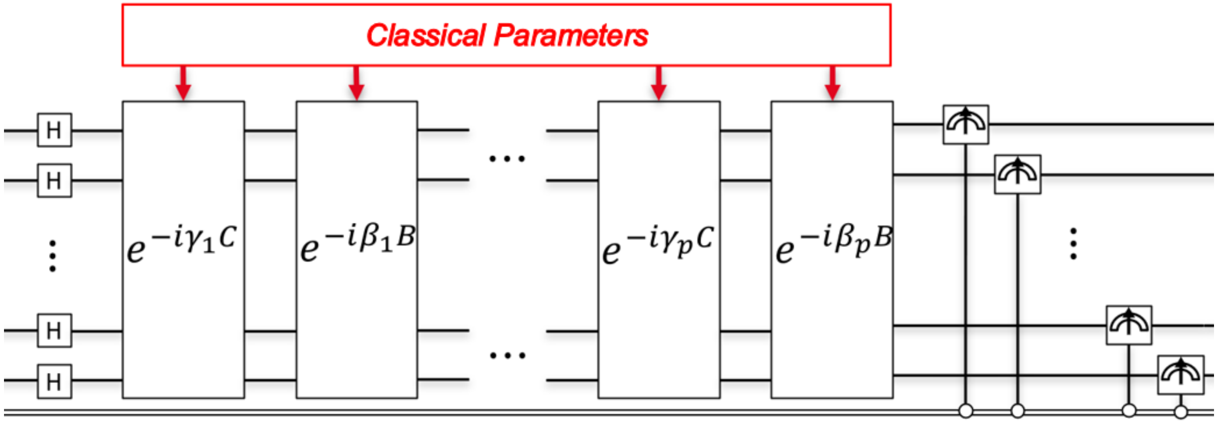


EX : PATH PLANNING WITH THREATS

QAOA

Combinatorial optimization problems may be specified by n bits and m clauses :

$$C(z) = \sum_\alpha C_\alpha(z) \text{ with } z = z_1 \ldots z_n \text{ a n bit string}$$



$$U(C,\gamma) = e^{-i\gamma C} = \prod_{\alpha=1}^{m} e^{-i\gamma C_\alpha}$$

$$U(B,\beta) = e^{-i\beta B} = \prod_{j=1}^{n} e^{-i\beta\sigma_j^x}$$

with $B = \sum_{j=1}^{n} \sigma_j^x$

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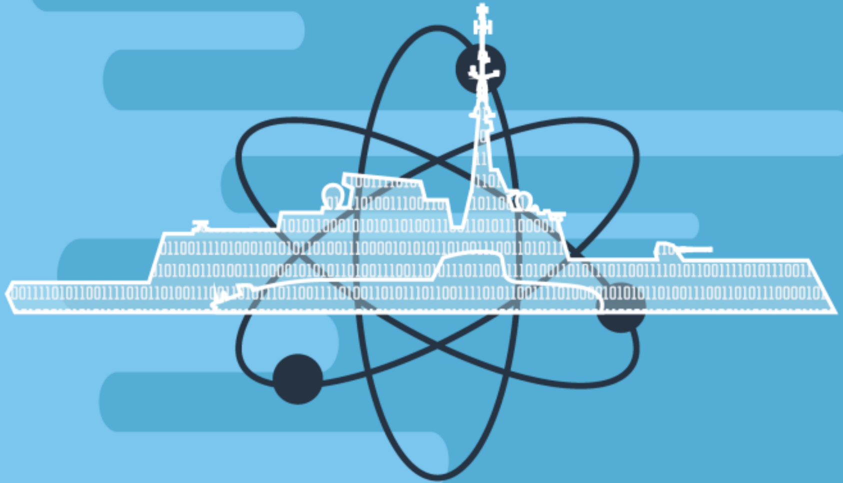
SOLVING A SHORTEST PATH PROBLEM WITH QAOA



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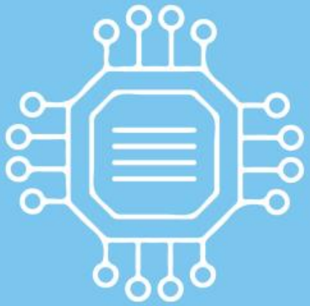
HOW TO APPLY QAOA

FOR EVERY COMBINATORIAL OPTIMIZATION PROBLEM

COST FUNCTION

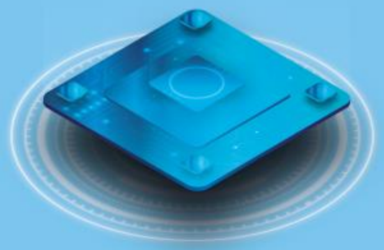
Optimizing variationnal parameters

COST HAMILTONIAN



Designing the quantum circuit

PROBABILITY DENSITY FROM THE CIRCUIT OUTPUT



Retrieveing the cost function input

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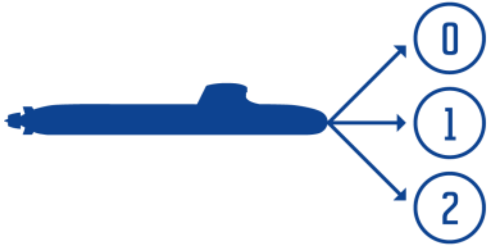
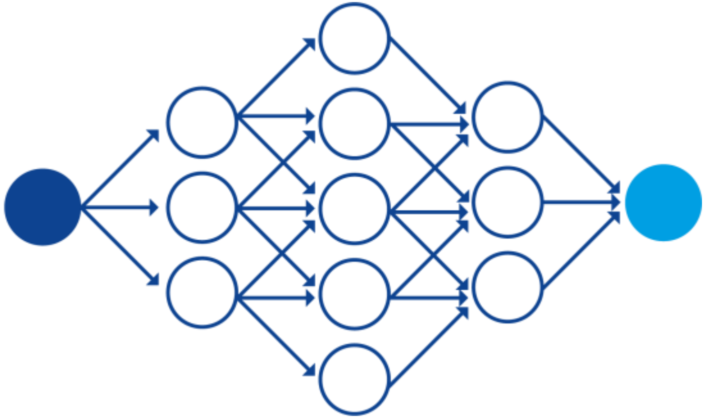
OUR SHORTEST PATH PROBLEM

One may consider a vehicule that can only move according to three directions:





A path can be expressed as a result matrix which each element represents a node:

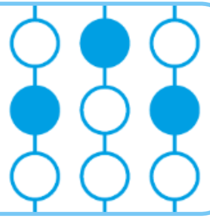
$$X = \begin{pmatrix} x_{1,1} & \cdots & x_{1,n} \\ \vdots & \cdots & \vdots \\ x_{n,1} & \cdots & x_{n,n} \end{pmatrix} \quad \text{avec} \quad x_{i,t} = \begin{cases} 1 \; if \; the \; node \; is \; used \; in \; the \; path \\ 0 \; unless \end{cases}$$

OUR SHORTEST PATH PROBLEM

- **Shortest path problem clauses:**

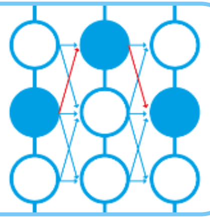
Clause 1 : A single node used in each column.

-> Every couple of active nodes in a same column is penalized.



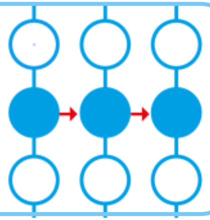
Clause 2 : All used nodes must be adjacent.

-> Every couple of active nodes non-adjacent is penalized.



Clause 3 : The overall distance must be minimized.

-> The distance between every active node is penalized.



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COST HAMILTONIAN

Building bricks of the cost hamiltonian:

$\boldsymbol{H_1} = \sigma_z^i$ and $\langle\psi|H_1|\psi\rangle = \langle\psi|I_d \otimes \sigma_z^i \otimes I_d|\psi\rangle = P(\overline{q_i}) - P(q_i)$

Minimizing $\langle\psi|H_1|\psi\rangle$ is equivalent to maximizing the states where the qubit q_i is measured in the state $|1\rangle$.

$\boldsymbol{H_2} = \sigma_z^i \sigma_z^j$ and $\langle\psi|H_2|\psi\rangle = \left\langle\psi\middle|I_d \otimes \sigma_z^i \otimes \sigma_z^j \otimes I_d\middle|\psi\right\rangle = P(\overline{q_i} \wedge \overline{q_j}) - P(\overline{q_i} \wedge q_j) - P(q_i \wedge \overline{q_j}) + P(q_i \wedge q_j)$

Minimizing $\langle\psi|H_2|\psi\rangle$ is equivalent to maximizing the states where $q_i \otimes q_j = 1$

$\boldsymbol{H_3} = \sigma_z^i + \sigma_z^j - \sigma_z^i \sigma_z^j$ and $\langle\psi|H_3|\psi\rangle = P(\overline{q_i}) - P(q_i) + P(\overline{q_j}) - P(q_j) - P(\overline{q_i} \wedge \overline{q_j}) + P(\overline{q_i} \wedge q_j) + P(q_i \wedge \overline{q_j}) - P(q_i \wedge q_j)$

Minimizing $\langle\psi|H_3|\psi\rangle$ is equivalent to maximizing the states where the qubits q_i and q_j are measured in the state $|1\rangle$.

By using wisely those building bricks, we may create many differents cost hamiltonian.

- **For our shortest path problem:**

Clause 1:

$$H_1 = h_1 \sum_{column} \sum_{(i,j) \in column} \sigma_z^i + \sigma_z^j - \sigma_z^i \sigma_z^j$$

Clause 2:

$$H_2 = h_2 \sum_{(i,j) \in Valid\_neighbours} \sigma_z^i + \sigma_z^j - \sigma_z^i \sigma_z^j$$

Clause 3 :

$$H_3 = h_3 \sum_{i \in qubits} weight(q_i) \times \sigma_z^i$$

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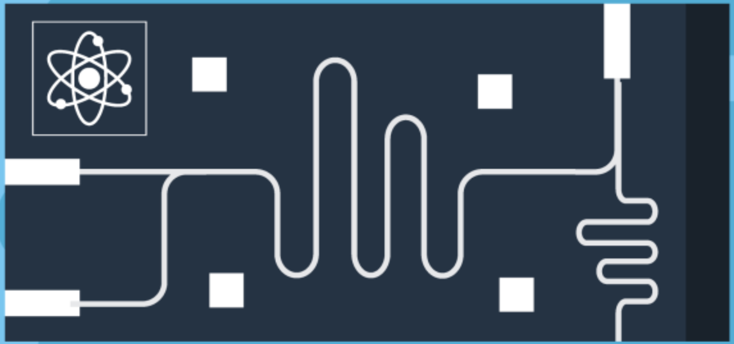
USING A HEURISTIC METHOD



ISSUES WITH THE QAOA

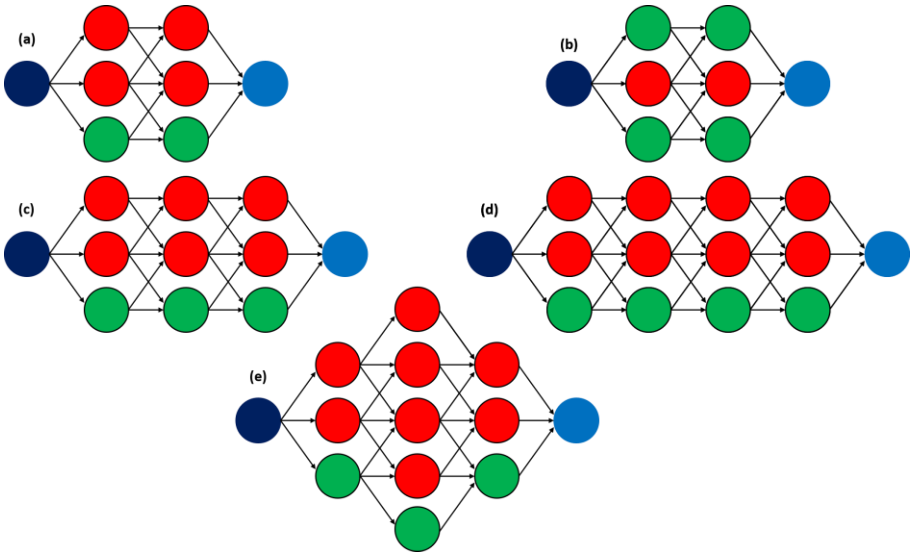
- **The number of iterations can be too high:**

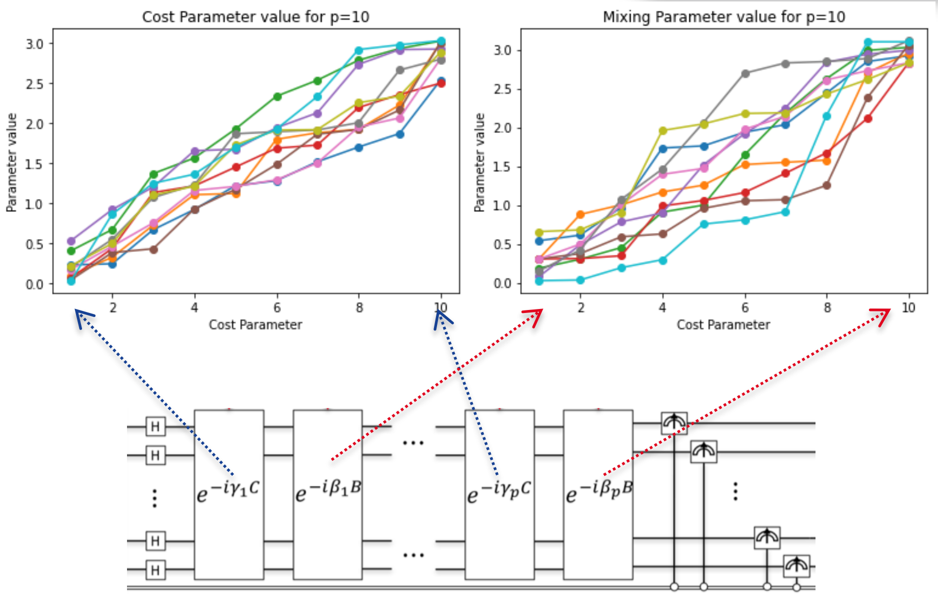
We may use a heuristic method in order to decrease the number of iterations.

- **The number of shots needed to find the probability density:**

For n qubits, there is 2^n possible solutions. It is important to start the search of optimal variationnal parameter with a probability density centered on few solutions.

PATTERN-BASED HEURISTIC METHOD





5 types of graphs (on the left) tested and the same pattern observed with the optimal variationnal parameters;

optimal variationnal parameters (on the right) for the graph (a).

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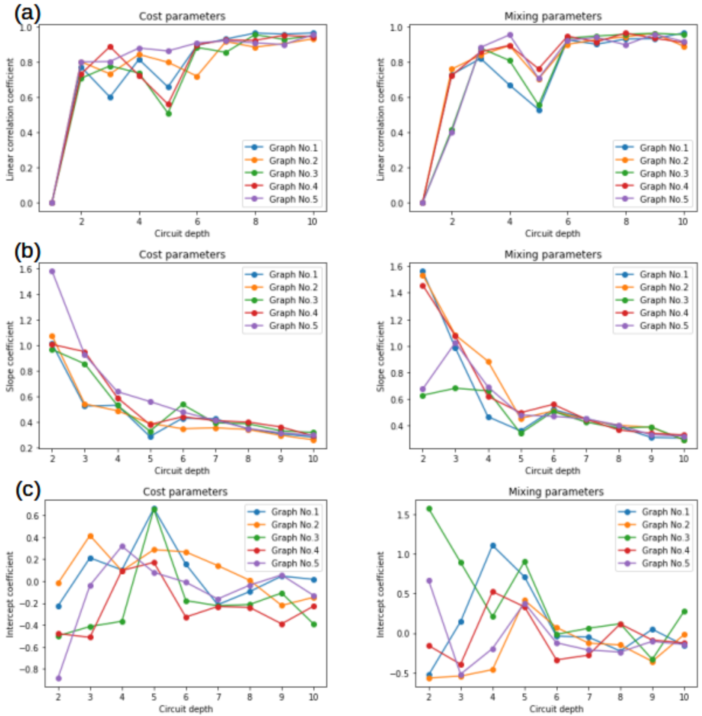
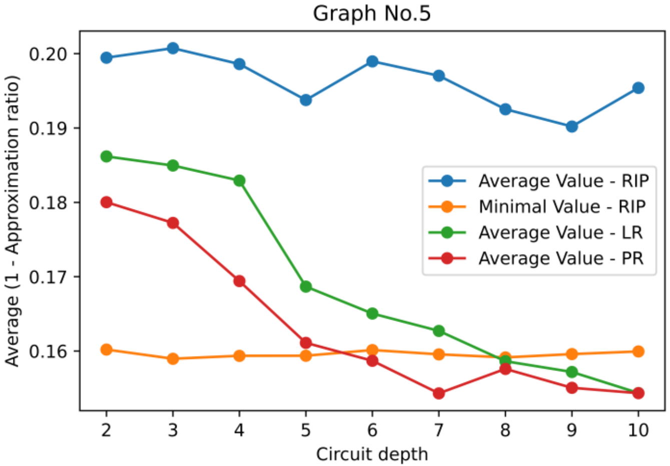
PATTERN-BASED HEURISTIC METHOD





Linear regression coefficient (on the left) and quality of the QAOA result while using random initial parameters (RIP) and heuristic method (LR for linear regression and PR for polynomial regression).

We used a linear regression to create an pattern based heuristic method to choose our initial parameters.

The number of iterations is linear with the circuit depth.

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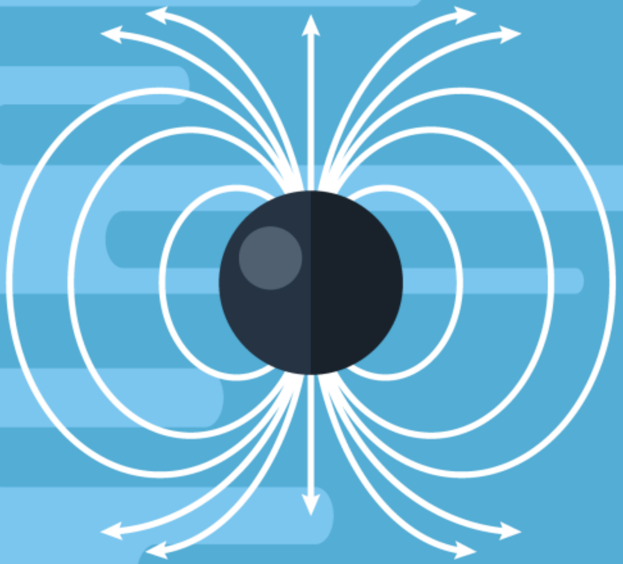
CONCLUSION



CONCLUSION

- Understanding the construction of hamiltonian allows to adapt the circuit to the problem we want to solve.
- The required iterations needed to solve such a problem does not apparently give a quantum speed-up.
- Working on the problem's symmetries and patterns, one can reduce the number of iterations.
- For similar graphs shapes, the optimal initial states can be fitted and reused to speed-up the computation.
- For differents sets of constraints, the optimal initial states can change and require a first full round of computations.
- And of course : the larger the graph is, the more qubits are needed : could be of interest if a middle sized graph has constraints making the problem hard to solve.